Completing the Calculation of BLM corrections to $\bar{B} \to X_s\gamma$

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Perturbative $\mathcal{O}(\alpha_s^2)$ corrections to $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the BLM approximation receive contributions from two-, three- and four-body final states. While all the two-body results are well established by now, the other ones have remained incomplete for several years. Here, we calculate the last contribution that has been missing to date, namely the one originating from interference of the current-current and gluonic dipole operators ($K_{18}^{(2)\beta_0}$ and $K_{28}^{(2)\beta_0}$). Moreover, we confirm all the previously known results for BLM corrections to the photon energy spectrum that involve the current-current operators (e.g., $K_{22}^{(2)\beta_0}$ and $K_{27}^{(2)\beta_0}$). Finally, we also confirm the recent findings of Ferroglia and Haisch on self-interference of the gluonic dipole operator ($K_{88}^{(2)\beta_0}$).

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I. INTRODUCTION

Weak radiative decay of the B meson is a well-known probe of physics beyond the Standard Model (SM). Calculations of its inclusive branching ratio in the SM for $E_\gamma > 1.6$ GeV give [1, 2]

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm SM} = (3.15 \pm 0.23) \times 10^{-4}, \tag{1}$$

which agrees within 1.2σ with the world average [3]

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm exp} = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}. \tag{2}$$

The above experimental result includes a model uncertainty that is due to averaging several measurements with various photon energy cuts E_0 and extrapolating them to $E_0 = 1.6$ GeV where the theory prediction is most reliable. Measurements with energy cuts $1.8~\text{GeV} \leq E_0 \leq 2.0$ GeV [4–6] have significantly smaller background-subtraction errors than those with $E_0 = 1.7$ GeV [4]. More work at balancing model-dependence and background-subtraction uncertainties is necessary in the future to obtain accurate experimental averages.

As far as the SM calculations are concerned, further improvements require another critical re-analysis of non-perturbative effects [7], as well as a full perturbative $\mathcal{O}(\alpha_s^2)$ evaluation of $\Gamma(b \to X_s^p\gamma)$, where X_s^p stands for s, sg and $sq\bar{q}$ partonic states ($q = u, d, s$). Such calculations are most conveniently performed in the framework of an effective low-energy theory that arises from the SM via decoupling of the W boson and all the heavier particles. So long as higher-order electroweak and/or CKM-suppressed effects are neglected, the relevant flavor-changing weak interactions at the renormalization scale $\mu_b \sim m_b/2$ are given by

$$\mathcal{L}_{\rm weak} = \frac{4G_F}{\sqrt{2}} \sum_{i=1}^{8} C_i(\mu_b) Q_i, \tag{3}$$

where Q_i denote either dipole-type or four-quark operators (see below), and $C_i(\mu_b)$ stand for their Wilson coefficients.

Following Refs. [2, 8], we shall normalize the radiative decay rate to the charmless semileptonic one, and parametrize their rato in terms of symmetric matrices $K_{ij}(\mu_b, E_0)$ as follows:

$$\frac{\Gamma[b \to X_s\gamma]}{\Gamma[b \to X_u e\bar{\nu}]} = \left|\frac{V_{ts}^* V_{tb}}{V_{ub}}\right|^2 \frac{6\alpha_{\rm em}}{\pi} \sum_{i,j=1}^{8} C_i^{\rm eff} C_j^{\rm eff} K_{ij}, \tag{4}$$

where $C_i^{\rm eff}$ are certain linear combinations of C_i, see Eq. (5) of Ref. [9]. Evaluation of all the $C_i^{\rm eff}(\mu_b)$ up to the Next-to-Next-to-Leading Order (NNLO) in QCD has been already completed several years ago [10].

In the perturbative expansion of K_{ij}

$$K_{ij} = K_{ij}^{(0)} + \frac{\alpha_s}{4\pi} K_{ij}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 K_{ij}^{(2)} + \ldots \tag{5}$$

all the $\mathcal{O}(1)$ and $\mathcal{O}(\alpha_s)$ terms are known since a long time [11]. As far as $K_{ij}^{(2)}$ are concerned, the so-called penguin four-quark operators $Q_3, \ldots, Q_6$ can be neglected thanks to smallness of their Wilson coefficients. We can restrict our attention to

$$\begin{aligned}
Q_1 &= (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L), \\
Q_2 &= (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L), \\
Q_7 &= \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \\
Q_8 &= \frac{g}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a,
\end{aligned} \tag{6}$$

i.e. consider $K_{ij}^{(2)}$ with $i, j \in \{1, 2, 7, 8\}$ only.

At present, $K_{ij}^{(2)}$ are known in a complete manner [12–14] for $(ij) = (77)$ and (78), while the other cases are estimated [15–17] using the BLM [18] approximation. In Ref. [2], non-BLM contributions to the decay rate have been calculated in the $m_c \gg m_b/2$ limit, and then interpolated downwards in m_c assuming that they vanish at $m_c = 0$. Such a treatment of non-BLM NNLO corrections in the evaluation of Eq. (1) still remains the current state-of-art for the numerically important but yet unknown $K_{17}^{(2)}$ and $K_{27}^{(2)}$.

(ij)	final state multiplicity	original calculation	confirmation
(77)	2	[16]	[12]
(77)	3, 4	[15]	[12, 16]
(78)	2	[16]	[13]
(78)	3, 4	[15]	[13, 17]
(88)	3, 4	[17]	this paper
(17),(27)	2	[16]	[19]
(17), (27)	3, 4	[15]	this paper
(11), (12), (22)	3, 4	[15]	this paper
(18), (28)	3, 4	this paper	

TABLE I: Present status of $K_{ij}^{(2)\beta_0}$ calculations.

The BLM and non-BLM contributions to $K_{ij}^{(2)}$ are denoted by $K_{ij}^{(2)\beta_0}$ and $K_{ij}^{(2)\mathrm{rem}}$, respectively. The latter are independent on n_l (the number of massless quark flavors), while the former are proportional to $\beta_0 = 11 - \frac{2}{3}(n_l + 2)$. In practice $n_l = 3$ because masses of the light $q = u, d, s$ quarks are neglected in loops on the gluon lines in $b \to s\gamma$ and $b \to sg\gamma$, as well as for external $q\bar{q}$ pairs in $b \to sg^*\gamma \to sq\bar{q}\gamma$. Although masses of the c and b quarks are not neglected, all the quantities in Eq. (5) are $\overline{\mathrm{MS}}$-renormalized at μ_b in the five-flavor theory, which justifies the use of five-flavor β_0 in $K_{ij}^{(2)\beta_0}$. Effects of non-zero values of m_c and m_b in loops on the gluon lines are known from Refs. [14, 19] for all the $K_{ij}^{(2)}$ with $i, j \in \{1, 2, 7, 8\}$. No real $c\bar{c}$ pair production is included in $b \to X_s^p\gamma$ by definition, while $b\bar{b}$ production is kinematically forbidden anyway.

Contributions to $K_{ij}^{(2)\beta_0}$ from the $b \to s\gamma$ channel arise for $(ij) =$ (17), (27), (77) and (78) only. They were originally calculated in Ref. [16]. Three- and four-body final state contributions ($b \to sg\gamma$ and $b \to sg^*\gamma \to sq\bar{q}\gamma$) for all the $i, j \in \{1, 2, 7\}$ cases and for $K_{78}^{(2)\beta_0}$ were evaluated first in Ref. [15]. Recently, $K_{88}^{(2)\beta_0}$ has been found by Ferroglia and Haisch [17].

In the present paper, we provide the last two missing contributions, namely $K_{18}^{(2)\beta_0}$ and $K_{28}^{(2)\beta_0}$. Moreover, we confirm the results for $(ij) =$ (11), (12), (22), (17) and (27) from Ref. [15], as well as for $K_{88}^{(2)\beta_0}$ from Ref. [17]. Table I summarizes the present status of $K_{ij}^{(2)\beta_0}$ calculations.

The article is organized as follows. In Sec. II, our evaluation of $K_{18}^{(2)\beta_0}$ and $K_{28}^{(2)\beta_0}$ is presented. Sec. III is devoted to the remaining contributions that involve the current-current operators (Q_1 and Q_2). Self-interference of the gluonic dipole operator Q_8 is considered in Sec. IV. We conclude in Sec. V.

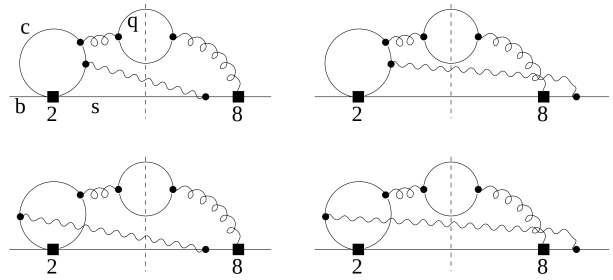

FIG. 1: Feynman diagrams that determine $K_{28}^{(2)\beta_0}$.

II. CALCULATION OF $K_{18}^{(2)\beta_0}$ AND $K_{28}^{(2)\beta_0}$

Determination of $K_{18}^{(2)\beta_0}$ and $K_{28}^{(2)\beta_0}$ amounts to evaluating n_l-dependent parts of $K_{18}^{(2)}$ and $K_{28}^{(2)}$. The latter originates from interference of decay amplitudes generated by the current-current operator Q_2 and the gluonic dipole operator Q_8. The contributing Feynman diagrams are most conveniently presented using Cutkosky rules [20] as four-loop propagator diagrams with unitarity cuts. They are displayed in Fig. 1. In dimensional regularization, no diagrams with cuts through the gluon lines need to be considered because the massless $q\bar{q}$-loop integral is scaleless for an on-shell gluon, which implies that all such diagrams vanish. If Q_2 is replaced by Q_1, the color factor gets modified according to $T^a \to T^bT^aT^b = -\frac{1}{6}T^a$, which leads to a simple relation

$$K_{18}^{(2)\beta_0} = -\frac{1}{6}K_{28}^{(2)\beta_0}. \tag{7}$$

Following the conventions introduced in Ref. [15], we exclude the diagrams depicted in Fig. 2 from the BLM approximation despite their n_l-dependence. They are correlated via renormalization group with tree-level $b \to sq\bar{q}\gamma$ matrix elements of the neglected four-quark operators Q_3, ..., Q_6. Excluding those diagrams from the BLM calculation is indeed reasonable. No other n_l-dependent diagrams arise because the Q_2-generated charm loops vanish if the on shell photon alone is emitted from them.

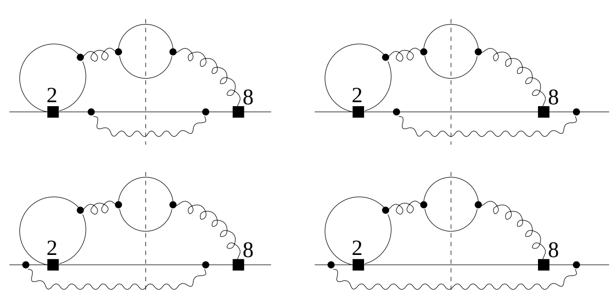

FIG. 2: Feynman diagrams excluded from $K_{28}^{(2)\beta_0}$.

In our actual evaluation of $K_{28}^{(2)\beta_0}$, the method of Smith and Voloshin [21] has been applied. It amounts to considering lower-order diagrams that are obtained from those in Fig. 1 by removing the $q\bar{q}$ loop from the gluon propagators. However, an arbitrary auxiliary mass of the gluon needs to be introduced. Next, integration over the gluon mass should be performed according to the formulae of Ref. [21].

We have carried out the calculation via direct integration over the 3-body partly massive phase space ($m_s = 0$) that is conveniently parametrized in terms of two variables: $u = 2(p_b p_\gamma)/m_b^2$ and $s = (p_g + p_\gamma)^2/m_b^2$. Explicit results from Sec. 4 of Ref. [22] for the one-loop Q_2 amplitude with an external off-shell gluon have appeared to be useful. Once the Dirac algebra is performed, we are left with precisely the same two Feynman parameter integrals as in Eqs. (4.21) and (4.22) of that paper, namely

$$\begin{aligned} F_b(s,z,v) &= \int_0^1 dx \int_0^1 dy \, \frac{1}{v + ys - \frac{z-i\varepsilon}{x(1-x)}}, \\ F_g(s,z,v) &= \int_0^1 dx \int_0^1 dy \, \frac{-xy}{v + ys - \frac{z-i\varepsilon}{x(1-x)}}, \end{aligned} \tag{8}$$

where $z = m_c^2/m_b^2$ and $v = m_{\rm gluon}^2/m_b^2$. Considering them here in $D = 4$ dimensions is sufficient because the calculation is free of ultraviolet, infrared or collinear divergences. Integrations over the two Feynman parameters x and y and over the phase-space variable u are performed analytically in a straightforward manner. The remaining two integrations (over s and v) have been completed numerically. More details will be presented elsewhere [23].

For $K_{ij}^{(2)\beta_0}$ with $i,j \neq 7$ we shall use the following notation (consistent with Ref. [2]):

$$K_{ij}^{(2)\beta_0} = 2(1+\delta_{ij})\beta_0 \left[\phi_{ij}^{(1)}(\delta) L_b + h_{ij}^{(2)}(\delta) \right], \tag{9}$$

where $\delta = 1 - 2E_0/m_b$, $L_b = \ln\left(\mu_b^2/m_b^2\right)$, and $\phi_{ij}^{(1)}(\delta)$ are the well-known NLO bremsstrahlung functions collected in Appendix E of Ref. [8].

Our final result for the function $h_{28}^{(2)}(\delta)$ reads

$$\begin{aligned} h_{28}^{(2)}(\delta) &= 0.02605 + 0.1679\,\delta - 0.1970\,\delta^2 \\ &+ (-0.03801 + 0.6017\,\delta - 0.7558\,\delta^2)\, z^{\frac{1}{2}} \\ &+ (2.755 - 10.03\,\delta + 11.27\,\delta^2)\, z \\ &+ (-27.05 + 68.47\,\delta - 72.51\,\delta^2)\, z^{\frac{3}{2}} \\ &+ (85.87 - 289.3\,\delta + 297.7\,\delta^2)\, z^2 \\ &+ (-91.53 + 399.8\,\delta - 399.9\,\delta^2)\, z^{\frac{5}{2}}. \end{aligned} \tag{10}$$

The above expression is a numerical fit that remains accurate in the ranges $0 \le z \le 0.13$ and $0.2 \le \delta \le 0.6$. These ranges will also be valid for the fits in Sec III. The central values used in Eq. (1) are $\delta = 1 - 2(1.6/4.68) \simeq 0.316$ and $z = [m_c(1.5\text{GeV})/4.68]^2 \simeq 0.0584$.

Eq. (10) is the main new result of the present paper. Its numerical effect on the branching ratio turns out to be miniscule (below 0.1%). However, the purpose of the present calculation is not finding sizeable effects but rather removing several minor uncertainties that had to be taken into account in Refs. [1, 2] in estimating the $\pm 3\%$ perturbative error that was unrelated to the m_c-interpolation.

III. OTHER CONTRIBUTIONS FROM CURRENT-CURRENT OPERATORS

Let us now consider $K_{ij}^{(2)\beta_0}$ for $(ij) \in \{(11),(12),(22)\}$. The three Feynman diagrams to be calculated in the (22) case are obtained from the left parts of the cut diagrams in Fig. 1 by forming all the possible interference terms. The cases (11) and (12) differ from (22) by color factors (analogously to Eq. (7)), namely

$$K_{22}^{(2)\beta_0} = -6 K_{12}^{(2)\beta_0} = 36 K_{11}^{(2)\beta_0}. \tag{11}$$

As before, only the diagrams with both the photon and the gluon coupled to the charm loop are included in the BLM approximation for the $b \to s g^* \gamma \to s q \bar{q} \gamma$ channel.

Using precisely the same methods as in Sec. II, we obtain the following numerical fit:

$$\begin{aligned} h_{22}^{(2)}(\delta) &= 0.01370 + 0.3357\,\delta - 0.08668\,\delta^2 \\ &+ (0.3575 + 1.825\,\delta - 0.3743\,\delta^2)\, z^{\frac{1}{2}} \\ &+ (-2.306 - 5.800\,\delta - 6.226\,\delta^2)\, z \\ &+ (3.449 - 0.5480\,\delta + 17.27\,\delta^2)\, z^{\frac{3}{2}}. \end{aligned} \tag{12}$$

Similarly, for the photonic dipole (Q_7) and the current-current operator interferences, we find

$$\begin{aligned} h_{27}^{(2)}(\delta) &= -0.1755 - 1.455\,\delta + 1.119\,\delta^2 \\ &+ (0.7260 - 7.230\,\delta + 5.977\,\delta^2)\, z^{\frac{1}{2}} \\ &+ (13.79 + 113.7\,\delta - 100.4\,\delta^2)\, z \\ &+ (-145.1 - 307.1\,\delta + 388.5\,\delta^2)\, z^{\frac{3}{2}} \\ &+ (475.2 + 313.0\,\delta - 775.8\,\delta^2)\, z^2 \\ &+ (-509.7 - 126.1\,\delta + 646.2\,\delta^2)\, z^{\frac{5}{2}}, \end{aligned} \tag{13}$$

together with $K_{17}^{(2)\beta_0} = -\frac{1}{6} K_{27}^{(2)\beta_0}$. However, the two-body contribution T is non-vanishing in this case, so instead of Eq. (9) one has

$$K_{27}^{(2)\beta_0} = T + 2\phi_{27}^{(2)\beta_0} \equiv T + 2\beta_0 \left[\phi_{27}^{(1)} L_b + h_{27}^{(2)} \right]. \tag{14}$$

Explicit formulae for T can be found in Ref. [2].

To compare our expressions in Eqs. (12) and (13) to Ref. [15], we made use of their results provided to us in the form of a numerical grid [24]. The grid described contributions to the differential photon energy spectrum in the ranges $0.2\, m_b \le E_\gamma \le m_b/2$ and $0 \le z \le 0.13$.

The originally published fits in Eq. (12) of Ref. [15] were valid in more narrow ranges, especially in the case of z. After performing an accurate fit to the grid, we integrated the spectrum over E_γ from E_0 to $m_b/2$, and then compared the outcome to our results for $h_{22}^{(2)}(\delta)$ and $h_{27}^{(2)}(\delta)$ in the ranges $0.2 \le \delta \le 0.6$ and $0 \le z \le 0.13$. A perfect agreement was found immediately at the first attempt to perform such a comparison, indicating undoubtedly that we can confirm the results of Ref. [15].

It is interesting to observe that $h_{22}^{(2)}$ affects the branching ratio by $+1.9\%$, which remains within the assumed $\pm 3\%$ uncertainty for all such effects in Eq. (1). Neither this contribution nor the smaller ones from $h_{27}^{(2)}$ and $h_{78}^{(2)}$ were included in the analysis of Refs. [1, 2].

IV. GLUONIC DIPOLE OPERATOR SELF-INTERFERENCE

The three Feynman diagrams that matter for $K_{88}^{(2)\beta_0}$ are obtained from the right parts of the cut diagrams in Fig. 1 by forming all the possible interference terms. Contrary to the previously discussed cases, a collinear divergence arises here, and a non-vanishing mass of the s-quark must be retained in the line from which the photon is emitted. In analogy to the NLO calculation of $K_{88}^{(1)}$, we shall keep this mass whenever it can produce $\ln(m_b/m_s)$, but neglect all the power corrections $(m_s/m_b)^n$.

Note that only $b \to sg^*\gamma \to sq\bar{q}\gamma$ matters for $K_{88}^{(2)\beta_0}$, which means that no photon emission from the $q\bar{q}$ pair needs to be considered. Amplitudes with such an emission would not be proportional to n_l but rather weighted with the quark electric charges. Consequently, the quarks into which the gluon fragments can be kept massless from the outset, even if they are the s quarks.

As the interfering amplitudes are tree-level here, all the phase-space integrals and the ficticious gluon mass integral can be performed analytically. We obtain

$$\begin{aligned} h_{88}^{(2)}(\delta) &= \tfrac{4}{27}\Big\{\big[\big(1+\tfrac{1}{2}\delta\big)\,\delta\ln\delta - 6\ln(1-\delta) - 2\mathrm{Li}_2(1-\delta) \\ &+ \tfrac{1}{3}\pi^2 - \tfrac{16}{3}\delta - \tfrac{5}{3}\delta^2 + \tfrac{1}{9}\delta^3\big]\ln\frac{m_b}{m_s} - 2\mathrm{Li}_3(\delta) \\ &+ (5-2\ln\delta)\left[\mathrm{Li}_2(1-\delta) - \tfrac{1}{6}\pi^2\right] - \tfrac{1}{12}\pi^2\delta\,(2+\delta) \\ &+ \left[\tfrac{1}{2}\delta + \tfrac{1}{4}\delta^2 - \ln(1-\delta)\right]\ln^2\delta + \left(\tfrac{151}{18} - \tfrac{1}{3}\pi^2\right)\times \\ &\times\ \ln(1-\delta) + \left(-\tfrac{53}{12} - \tfrac{19}{12}\delta + \tfrac{2}{9}\delta^2\right)\delta\ln\delta \\ &+ \tfrac{787}{72}\delta + \tfrac{227}{72}\delta^2 - \tfrac{41}{72}\delta^3\Big\}\,. \qquad (15) \end{aligned}$$

The corresponding contribution to the photon energy spectrum is found by differentiating $K_{88}^{(2)\beta_0}$ with respect to δ. Doing so, we find perfect agreement with the very recent article of Ferroglia and Haisch [17]. An extended discussion of collinear divergences can be found there, which adds new elements to the previous analyses in Refs. [7, 25]. Replacing the perturbative collinear regulator m_s by a physical hadronic one can hardly be performed in a quantitatively precise manner given our poor knowledge of the QCD bound state properties. Fortunately, the gluonic dipole operator self-interference undergoes significant suppression in $\bar{B} \to X_s\gamma$ due to $(Q_d C_8/C_7)^2 \simeq 1/36$, as well as the relatively high photon energy cut $E_0 \sim m_b/3$. Evaluation of $K_{88}^{(2)\beta_0}$ provides just a check that no unexpected large numerical factors overcome this suppression at the perturbative level. The overall effect of $K_{88}^{(2)\beta_0}$ on the $b \to X_s^p\gamma$ decay width does not exceed 0.2%.

V. CONCLUSIONS

The NNLO QCD corrections to $b \to X_s^p\gamma$ in the BLM approximation receive contributions from $b \to s\gamma$, $b \to sg\gamma$ and $b \to sg^*\gamma \to sq\bar{q}\gamma$. The former results are well established by now, while the $b \to sg\gamma$ BLM amplitudes vanish in dimensional regularization. In this article, we have calculated the last missing $b \to sg^*\gamma \to sq\bar{q}\gamma$ contributions, namely $K_{18}^{(2)\beta_0}$ and $K_{28}^{(2)\beta_0}$. In addition, we have confirmed all the previously known results for BLM corrections to the photon energy spectrum that involve the current-current operators, as well as the recently found $K_{88}^{(2)\beta_0}$. Numerical effects of all these quantities on the branching ratio remain within the $\pm 3\%$ perturbative uncertainty estimated in Refs. [1, 2].

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